Regulating a Monopolist with Unverifiable Quality

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Introduction

- The PPACA (Healthcare Reform Bill) includes provisions for changing the way Medicare pays for health services:

    1. Provides funding for the CMS to design, test, and implement innovative payment and delivery arrangements designed to improve quality and reduce cost of health care.
    2. Provides incentives for the formation of Accountable Care Organizations.

- Traditional Medicare and Medicaid payment arrangements are based on DRG prospective payments, but are designed to represent some measure of average costs with adjustments for region and patient mix indices.

- Previous research has shown that a prospective payment system will reduce the cost of care relative to FFS.

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- However, one of the tradeoffs appear to be the quality of care.
- How should these new payment systems be designed?

Introduction

- The PPACA approaches regulating health insurers similarly to public utilities.
  - Regulates the rate-making and degree of coverage for insurers wanting to join health exchanges.
- *Large* body of literature and experience with the regulation of utilities and procurement.
- Not clear that the existing approaches to regulating quality are appropriate to the health services markets.
  - Minimum Service Quality
  - Rate of Return

Introduction

- Another approach to dealing with quality is to make it verifiable/quantifiable.
- HHS examines insurers and gives them a score based on a 5-star scale.
- Many states create hospital report cards.
    - Increases demand-elasticity to quality
    - Can contract on "grade"
- The NCLB Act is also an attempt to make quality verifiable.

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- Many states create hospital report cards.
  - Increases demand-elasticity to quality
  - Can contract on “grade”
- The NCLB Act is also an attempt to make quality verifiable.
- What is the value of making quality verifiable?
- What are the measures of quality that consumers value?

Regulatory Environment

To explore the nature of the regulator's problem in a market environment as challenging as that for health services I characterize the market as follows:

- The regulator has *minimal* information.
  - Can only monitor the price charged to consumers (if consumers are responsible for payment).
  - No contractable observation of cost, quantity, or quality
  - Can be thought of as representing a stress test of regulatory capabilities

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  - No contractable observation of cost, quantity, or quality
  - Can be thought of as representing a stress test of regulatory capabilities
- Consumer demand is responsive to the quality level.
  - Market demand may not be too sensitive to quality, but in a monopolistically competitive market, residual demand can be much more sensitive.
  - Without demand for quality it can only be induced through altruistic motives.

Regulatory Environment, con't.

- The firm knows more about either the cost or the demand for its good.
- Regulator acts as Stackelberg leader and designs a payment policy optimizing total social surplus given the strategic/incentive responses of both the hospitals and consumers.
- Regulator can make consumers price-insensitive by compensating the firm directly.
  - Marketed good: Consumers pay directly
  - Nonmarketed good: Regulator pays on behalf of consumers
- Additionally, in health services markets, firms exhibit a mix of objectives:
  - for-profit: profit-maximization
  - not-for-profit: ?

Questions

- What does the optimal payment policy look like and how does the firm's informational advantage distort the outcome in this setting?
  - Moral Hazard is not sufficient condition for distortion from social optimum (Caillaud, Guesnerie and Laffont; 1988).
  - Unverifiable quality is a dimension of moral hazard that interacts with consumer behavior.
- The strategic response of the firm and consumers clearly matter- how do they contribute to the distortion away from the socially-optimal outcome?
- How does the source of asymmetric information affect what can be achieved?
  - Classic difference in results between Baron and Myerson (1981) and Lewis and Sappington (1992)

Summary of Findings

- Information asymmetries lead to outcome distortions.
  - Marginal optimizer uses informational advantage to extract rents, average cost optimizer uses informational advantage to over produce
  - Distortion may be an *under-* or *oversupply* of the good... or *both*.
  - Direction of distortion cannot be known a priori.
  - Distortion depends on how the firm's cost of production changes with the state of the world as well as a complex interaction of quality and price elasticities.
    - Firm can have stronger preference for quality$\rightarrow$regulator will have to permit firm to "over cook" quality for incentive compatibility

- Nature of the payment rule changes with the objective of the firm.
  - Regulator can use both policy instruments to induce the desired level of output, *but* it still cannot induce the first-best outcome with a marketed good.
  - When the good is nonmarketed and the firm is a pure output-maximizer, the firm's output is a function of both policy instruments and its informational advantage can be eliminated

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  ...but with a social cost of using public funds.

Related Literature

- Asymmetric Demand, without Quality
  - Riordan (1984), Lewis and Sappington (1988 AER), Lewis and Sappington (1988 Rand), Aguirre and Beitia (2004)
- Symmetric Demand with Quality
  - Ma (1994), Rogerson (1994), Chalkley and Malcomson (1999 EJ)
- Symmetric Demand and Unknown Cost
  - Baron and Myerson (1982), Guesnerie and Laffont (1984), Laffont and Tirole (1986)
- Asymmetric Demand with Quality:
  - Lewis and Sappington (1992), Laffont and Tirole (1993)
- Altruistic Objective
  - Ellison and McGuire (1982), Chalkley and Malcomson (1999 JEH), Jack (2004)

Statement of the Model: Consumers

- Quality: $q \in \mathbb{R}_+$
- Price: $p \in \mathbb{R}_+$
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- Quality Demand: $q(p, x)$
  - $q_x > 0$, $q_p > 0$
- Consumer Benefit: $B(x, q)$
  - $B(x, q) = \int_0^x P(\tilde{x}, q) d\tilde{x}$
  - $B_x > 0$, $B_q > 0$, $B_{xx} \leq 0$, and $B_{qq} \leq 0$

Statement of the Model: Firm

- Produces output x with quality q
- Cost state $\theta \in [\underline{\theta}, \overline{\theta}] = \Theta$
  - Regulator's uncertainty: CDF F, PDF $f > 0$
- Cost of production: $c(x, q; \theta)$
  - $c_x > 0$, $c_q > 0$, $c_\theta > 0$
  - $c_{xx} \geq 0$, $c_{qq} \geq 0$, $c_{xq} \geq 0$, $c_{x\theta} > 0$, $c_{q\theta} \geq 0$, $\mathcal{C}^2$, quasiconcave
- Firm value for community benefit: $\phi(x, q)$
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- Firm's objective:

$$U(\Pi, \phi) = \beta\Pi + (1 - \beta)\phi, \quad \beta \in [0, 1]$$

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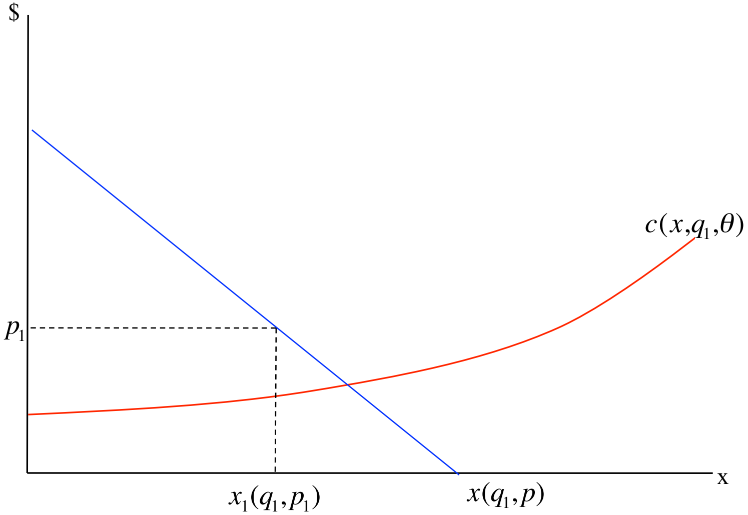
- $\dfrac{d^2 g}{dx^2} > 0$

Quality-Adjusted Cost



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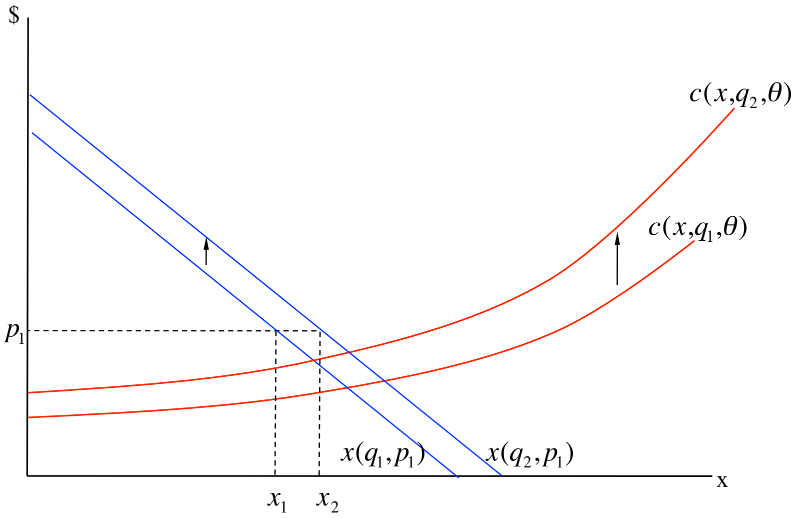

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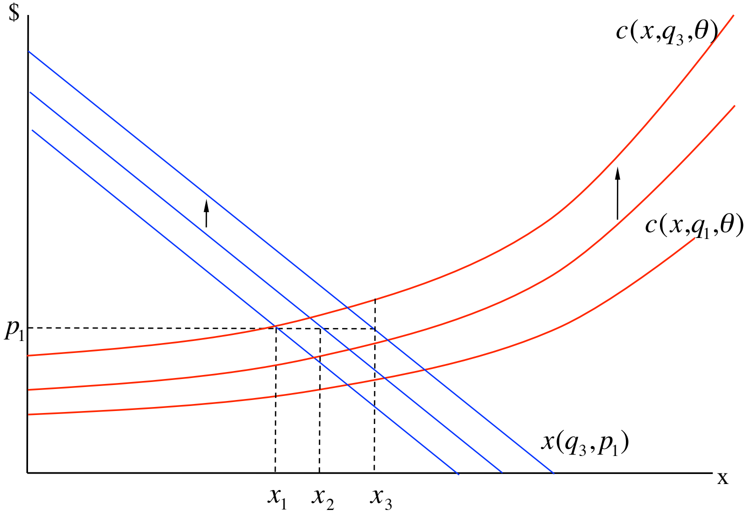

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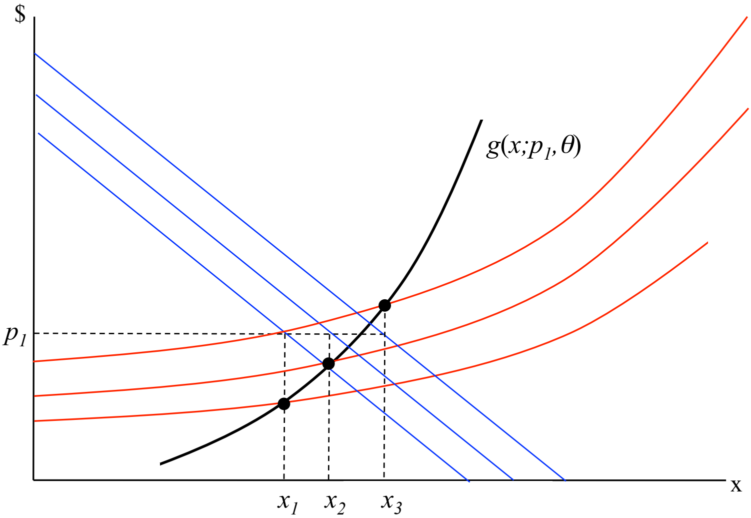

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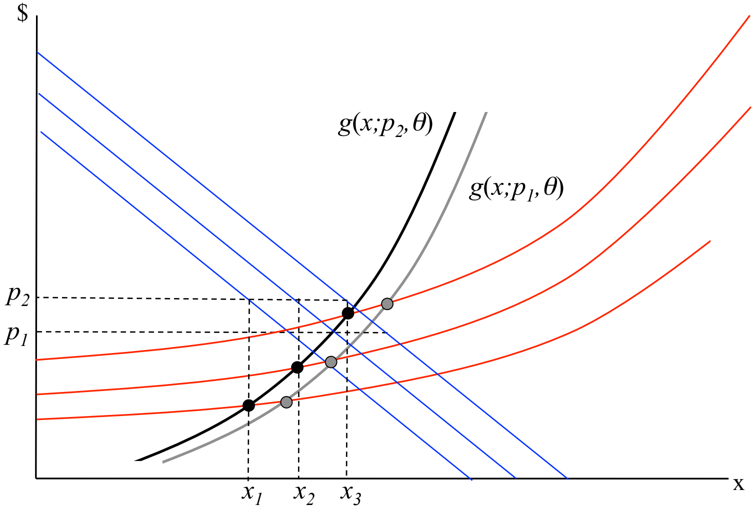

Model, con't

- Define *Quality-Adjusted* benefit: $V(x;p) = B(x, q(x,p))$
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- Define *Quality-Adjusted* benefit: $V(x;p) = B(x, q(x,p))$
  - $V_x > 0$, $V_{xx} < 0$, $V_p > 0$, $\mathcal{C}^2$
- Regulator: Stackelberg Leader
  - Contract: $\{p(\theta), T(\theta)\}_{\theta \in \Theta}$
    - p: unit payment
    - T: lump-sum transfer
  - Social cost of public funds: $\gamma > 0$ (with nonmarketed good only)

Model, con't

- The regulator's objective is to maximize a weighted sum of consumer and producer surplus:

$$\max_{p,T} \alpha CS + (1-\alpha)\Pi,$$

where

- CS = $V(x;p,\theta) - (px + T)$
- $\Pi = px - g(x;p,\theta) + T$
- $\alpha > 1/2$

Baseline: Social Optimum

- The regulator's problem can be expressed as

$$\max_{x,p,\Pi} V(x;p,\theta) - g(x;p,\theta) - \lambda\Pi \text{ s.t. } \Pi \geq 0,$$

where $\lambda = (2\alpha - 1)/\alpha$.

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Definition

The socially optimal outcome consists of the quantity, x^{so}, and prices, $\{p^{so}, T^{so}\}$ satisfying the conditions:

$$V_x = g_x,$$
$$V_p = g_p,$$
$$\Pi = 0.$$

- Given a contract $\{p, T\}$, firm chooses output, x^*, where

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- Firm's FOC implies 2 cases:
  1. Marginal Optimizer: Firm's preference leaves it with some positive profit.

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  2. Pure Output-maximizer: The firm's preference leaves it with zero profit.

$$px + T = g\left(x; p, \theta\right)$$
$$\Rightarrow x^* = x^*(p, T, \theta).$$

- Regulator's problem

$$\max_{p,\Pi} \; V\big(x^*(p,\theta);p\big) - g\big(x^*(p,\theta);p,\theta\big) - \lambda\Pi$$

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$$\frac{dV\left(x^*(p^{fb},\theta);p^{fb}\right)}{dp} = \frac{dg\left(x^*(p^{fb},\theta);p^{fb},\theta\right)}{dp}$$

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$$\frac{dV\left(x^*(p^{fb},\theta);p^{fb}\right)}{dp} = \frac{dg\left(x^*(p^{fb},\theta);p^{fb},\theta\right)}{dp}$$

$$V_x\left(x^*(p^{fb},\theta);p^{fb}\right)\tfrac{dx^*}{dp}(p^{fb},\theta) + V_p\left(x^*(p^{fb},\theta);p^{fb}\right) = g_x\left(x^*(p^{fb},\theta);p^{fb},\theta\right)\tfrac{dx^*}{dp}(p^{fb},\theta) + g_p\left(x^*(p^{fb},\theta);p^{fb},\theta\right).$$

Proposition

The optimal payment rule with symmetric cost information for a mixed-objective firm consists of the unique unit price $p^{fb}(\theta)$ and transfer payment $T^{fb}(\theta)$ satisfying:

$$p^{fb}(\theta) = V_x\left(x^{fb}; p^{fb}\right) + \frac{V_p\left(x^{fb}; p^{fb}(\theta)\right) - g_p\left(x^{fb}; p^{fb}(\theta), \theta\right)}{dx^*\left(p^{fb}(\theta), \theta\right)/dp} - \frac{(1-\beta)}{\beta}\phi'\left(x^{fb}(\theta)\right)$$

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and for a pure output-maximizing firm

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where $p(\theta) \leq g_x\left(x^{fb}(\theta); p^{fb}(\theta), \theta\right)$ for all $\theta \in \Theta$.

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Proposition

Given a price, p, output (and quality) may be over- or undersupplied relative to the socially-preferred level. The output differs from the socially preferred output according to the rule:

$$x^* \gtreqqless \arg\max_x (V - g) \text{ when } \; p \gtreqqless V_x(x^*; p) - \frac{(1-\beta)}{\beta}\phi'.$$

Standard screening problem, requires type-separation.

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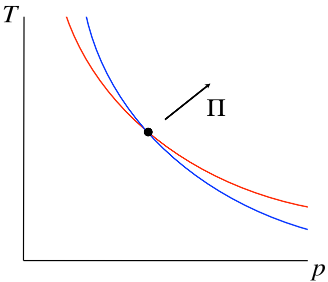
Definition

The single-crossing property holds if the firm's marginal rate of substitution (MRS) of price for transfer payment (U_p/U_T) is monotonic in θ for all $\theta \in \Theta$.



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Regulator's problem is now defined as:

$$\max_{p(\theta),\mathcal{U}(\theta)} \int_\Theta \left\{V\left(x^*(p(\theta),\theta);p(\theta)\right) - g\left(x^*(p(\theta),\theta);p(\theta),\theta\right) - \frac{\lambda}{\beta}\left(\mathcal{U}(\theta) - (1-\beta)\varphi(\theta)\right)\right\} dF(\theta)$$

where $\mathcal{U}(\theta) = \mathcal{U}(\theta,\theta)$, and $\varphi(\theta) = \phi(x^*(p(\theta),\theta))$.

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where $\mathcal{U}(\theta) = \mathcal{U}(\theta,\theta)$, and $\varphi(\theta) = \phi(x^*(p(\theta),\theta))$.
Subject to

$\mathcal{U}(\theta) \geq 0$	$\forall \theta \in \Theta$	Individual Rationality
$\mathcal{U}(\theta,\theta) \geq \mathcal{U}(\hat{\theta},\theta)$	$\forall \hat{\theta},\theta \in \Theta$	Incentive Compatibility

Lemma

The menu of two-part tariffs $\{p(\theta), T(\theta)\}_{\theta\in\Theta}$ *is incentive compatible if and only if it satisfies the conditions*

(i) $$\frac{d\mathcal{U}(\theta)}{d\theta} = \frac{\partial\mathcal{U}(\theta)}{\partial\theta} = \begin{cases} -\beta g_\theta\big(x^*(p(\theta),\theta);p(\theta),\theta\big) & \textit{when } \Pi(x^*) > 0, \\ & \end{cases}$$

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(ii) $sign[dp/d\theta] = sign\left[\frac{\partial}{\partial\theta}(U_p/U_T)\right]$.

- Necessary Cond.: $\frac{d\mathcal{U}}{d\theta}(\hat\theta(\theta),\theta) = \frac{\partial U}{\partial\hat\theta}\frac{\partial\hat\theta}{\partial\theta} + \frac{\partial U}{\partial x^*}\frac{\partial x^*}{\partial\theta} - \frac{\partial g}{\partial\theta} = 0$
- Sufficient Cond.: $\frac{\partial\mathcal{U}(\hat\theta,\theta)}{\partial\hat\theta} = \frac{dp}{d\hat\theta}\frac{\partial\{U_p/U_T\}(\tilde\theta)}{\partial\theta}(\theta-\hat\theta) \le 0$

Must account for the two cases:

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2. Pure output-maximizer: Firm's objective leaves it with zero rents.
    - Firm solves part of the regulator's problem by choosing the output that eliminates its rents
    - Regulator must design the policy so the firm does not misreport to increase output

The firm's value function can be expressed as:

$$\begin{aligned}\mathcal{U}(\theta) &= \mathcal{U}(\overline{\theta}) + \beta \int_{\theta}^{\overline{\theta}} g_\theta(x^*; p, \theta) d\theta \\ &= (1-\beta)\varphi(\overline{\theta}) + \beta \int_{\underline{\theta}}^{\overline{\theta}} g_\theta(x^*; p, \theta) d\theta.\end{aligned}$$

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Substituting $\mathcal{U}(\theta) = \beta\Pi(\theta) + (1-\beta)\varphi(\theta)$, rearranging, and taking expectations gives

$$E\Pi(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \tfrac{1-\beta}{\beta} \left(\varphi(\overline{\theta}) - \varphi(\theta)\right) + \int_{\theta}^{\overline{\theta}} g_\theta(x^*; p, \theta) d\theta \right\} dF(\theta).$$

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Corollary

An incentive compatible menu of two-part tariffs $\{p(\theta), T(\theta)\}_{\theta \in \Theta}$ *must leave a marginal optimizer with expected information rents:*

$$E\Pi(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \tfrac{1-\beta}{\beta} \left[\varphi(\overline{\theta}) - \phi\big(x^*(\theta)\big)\right] + \tfrac{F(\theta)}{f(\theta)} g_\theta(x^*; p, \theta) \right\} dF(\theta).$$

Regulator's problem can be solved using optimal control

$$H = \Big\{V\big(x^*(p(\theta),\theta);p(\theta)\big) - g\big(x^*(p(\theta),\theta);p(\theta),\theta\big) - \tfrac{\lambda}{\beta}\big(\mathcal{U}(\theta) - (1-\beta)\varphi(\theta)\big)\Big\}f(\theta)$$
$$- \delta(\theta)\beta g_\theta\big(x^*(p(\theta),\theta);p(\theta),\theta\big),$$

- State variable: $\mathcal{U}(\theta)$
- Control: $p(\theta)$
- Co-state variable: $\delta(\theta)$

Plugging in $\delta(\theta)$ and taking the first-order condition of the Hamiltonian yields:

$$\frac{d}{dp}\left\{V\left(x^*(p(\theta),\theta);p(\theta)\right)-g\left(x^*(p(\theta),\theta);p(\theta),\theta\right)\right\} = \lambda\left[\frac{F(\theta)}{f(\theta)}\frac{d}{dp}\{g_\theta(x^*;p,\theta)\} - \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}\right].$$

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Payment rule can be defined as the $\{p^{sb}(\theta),T^{sb}(\theta)\}_{\theta\in\Theta}$ solving

$$p^{sb}(\theta) = V_x\big(x^*;p^{sb}(\theta)\big) + \frac{V_p(\cdot)-g_p(\cdot)}{dx^*/dp} - \frac{1-\beta}{\beta}\phi'\big(x^{sb}(\theta)\big) - \frac{\lambda\frac{d}{dp}\{U(\theta)\}}{dx^*/dp},$$

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for all $\theta\in\Theta$.

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for all $\theta\in\Theta$.

1. When does the payment rule satisfy incentive compatibility?
2. What is the magnitude of the second-best payment relative to first-best?

Proposition

The relative magnitude of the second- to the first-best unit price is inversely related to the effect a price change has on the firm's information rent:

$$p^{sb} \begin{cases} < p^{fb} & \text{when } \frac{F(\theta)}{f(\theta)} d\{g_\theta(x^*;p,\theta)\}/dp > \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}, \\ = p^{fb} & \text{when } \frac{F(\theta)}{f(\theta)} d\{g_\theta(x^*;p,\theta)\}/dp = \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}, \\ > p^{fb} & \text{when } \frac{F(\theta)}{f(\theta)} d\{g_\theta(x^*;p,\theta)\}/dp < \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}. \end{cases}$$

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- $d\{V-g\}/dp > 0$
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Relative payments can vary, how does that affect output?

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$$\frac{d\mathcal{U}(\theta)}{dp} = \frac{d\Pi(\theta)}{dp} = \frac{d}{dp}\left\{\frac{F(\theta)}{f(\theta)}\frac{\partial g(x^*;p,\theta)}{\partial\theta}\right\}$$

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Decompose g_θ into its constituent parts:

$$\frac{d}{dp}\left\{\frac{\partial g}{\partial\theta}\right\} = \frac{\partial^2 g}{\partial\theta\partial p} + \frac{\partial^2 g}{\partial\theta\partial x}\frac{dx^*}{dp}$$

Relative Output

Marketed Good→ Pure Profit Maximizer

$$\frac{d\{g_\theta(x^*;p,\theta)\}}{dp} = g_{\theta p}(x^*;p,\theta) + g_{\theta x}(x^*;p,\theta)\frac{dx^*}{dp}$$

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Cross-partials

$$\frac{\partial^2 g(x;p,\theta)}{\partial\theta\partial p} = \frac{\partial^2 c\big(x,q(x,p);\theta\big)}{\partial\theta\partial q}\frac{dq(x,p)}{dp} \geq 0$$

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$$\frac{dx^*}{dp}$$

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$sign[d\{g_\theta(x^*;p,\theta)\}/dp]$ depends on sign and magnitude of

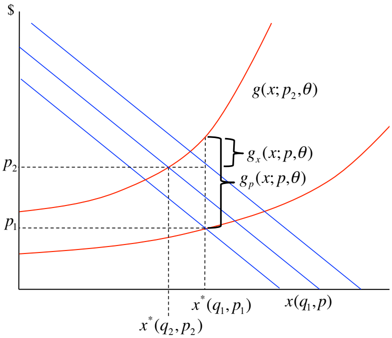
$$\frac{dx^*}{dp}$$

which depends on relative demand and cost-elasticities to quality:

$$\frac{d}{dp}\big(\epsilon_{x,q}px - \epsilon_{g,q}g(\cdot)\big) \lesseqgtr 0$$

Relative Output

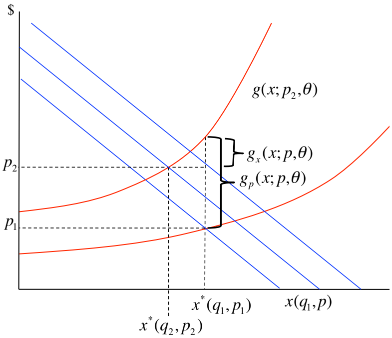
Marketed Good→ Pure Profit Maximizer



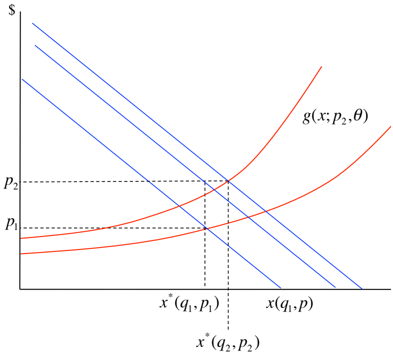
(a)

Relative Output

Marketed Good → Pure Profit Maximizer

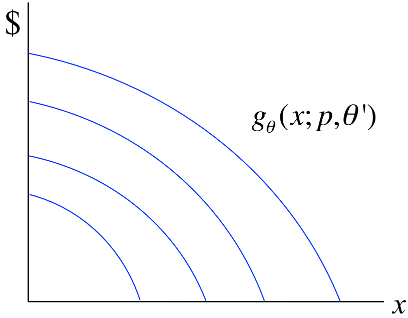


(c)



(d)

$$\frac{d\{g_\theta(x^*;p,\theta)\}}{dp} = g_{\theta p}(x^*;p,\theta) + g_{\theta x}(x^*;p,\theta)\frac{dx^*}{dp}$$



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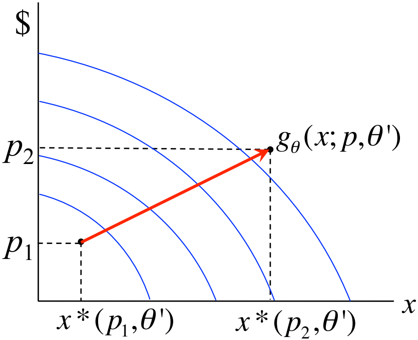

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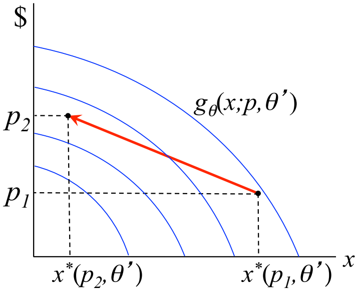
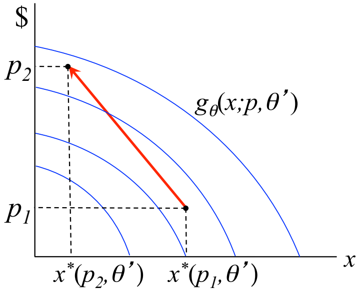


(e)



(f)

Proposition

With asymmetric cost information and a pure profit-maximizing firm, the relative size of the second- to first-best price and output is determined by the rules:

$$\text{(i)}\quad \frac{dx^*}{dp} < -\frac{g_{\theta p}}{g_{\theta x}} < 0 \quad \Rightarrow \frac{d}{dp}\left[\frac{\partial U}{\partial \theta}\right] < 0 \Rightarrow \begin{cases} p^{sb}(\theta) > p^{fb}(\theta) \\ x^{sb}(\theta) < x^{fb}(\theta) \end{cases}$$

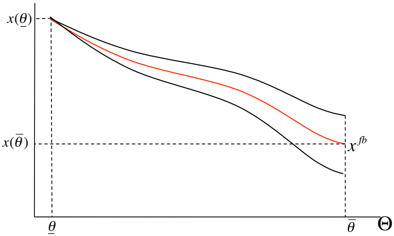
$$\text{(ii)}\quad -\frac{g_{\theta p}}{g_{\theta x}} < \frac{dx^*}{dp} < 0 \quad \Rightarrow \frac{d}{dp}\left[\frac{\partial U}{\partial \theta}\right] > 0 \Rightarrow \begin{cases} p^{sb}(\theta) < p^{fb}(\theta) \\ x^{sb}(\theta) > x^{fb}(\theta) \end{cases}$$

$$\text{(iii)}\quad -\frac{g_{\theta p}}{g_{\theta x}} < 0 < \frac{dx^*}{dp} \quad \Rightarrow \frac{d}{dp}\left[\frac{\partial U}{\partial \theta}\right] > 0 \Rightarrow \begin{cases} p^{sb}(\theta) < p^{fb}(\theta) \\ x^{sb}(\theta) < x^{fb}(\theta) \end{cases}$$

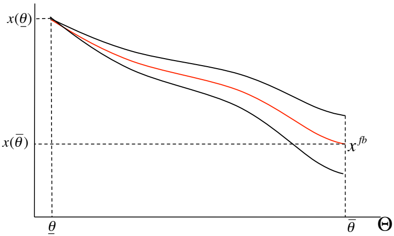
for any $\theta \in (\underline{\theta}, \overline{\theta}]$ *and at* $\underline{\theta}$, $p^{sb}(\underline{\theta}) = p^{fb}(\underline{\theta})$ *and* $x^{sb}(\underline{\theta}) = x^{fb}(\underline{\theta})$.

Relative Output

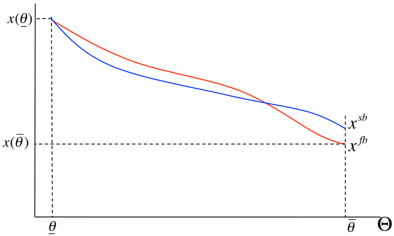
Marketed Good→ Pure Profit Maximizer



(g)


$x(\underline{\theta})$
$x(\overline{\theta})$
x^{fb}
$\underline{\theta}$
$\overline{\theta}$
Θ

(i)


$x(\underline{\theta})$
$x(\overline{\theta})$
x^{sb}
x^{fb}
$\underline{\theta}$
$\overline{\theta}$
Θ

(j)

Returning to the regulator's problem when the firm is a pure output maximizer:

- State variable: $S(\theta) \equiv V(x; p, \theta) - g(x; p, \theta)$
- Control: $p(\theta) = p^{np}(\theta)$
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The Hamiltonian is defined as:

$$H = S(\theta) + \delta(\theta)\Big\{\big(V_x - g_x\big)\frac{dx^*}{d\theta} + \Big(V_p - g_p + (V_x - g_x)\frac{dx^*}{dp}\Big)\frac{dp^{np}}{d\theta} - g_\theta\Big\}.$$

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The FOC of the Hamiltonian yields:

$$\frac{d}{dp}\Big\{\big(V_x - g_x\big)\frac{dx^*}{d\theta} + \Big(V_p - g_p + (V_x - g_x)\frac{dx^*}{dp}\Big)\frac{dp^{np}}{d\theta} - g_\theta\Big\} = 0$$

Recall from Lemma 1:

$$\frac{dx^*}{d\theta} = \frac{\partial x^*}{\partial \theta} = \frac{-g_\theta(x^*; p, \theta)}{g_x(x^*; p, \theta) - p} < 0$$

Recall from Lemma 1:

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The firm's "value" function can be expressed as:

$$x^*\left(\hat{\theta}(\theta), \theta\right) = x^*\left(\hat{\theta}(\overline{\theta}), \overline{\theta}\right) - \int_{\theta}^{\overline{\theta}} \frac{\partial x^*}{\partial \theta}\left(\hat{\theta}(\tilde{\theta}), \tilde{\theta}\right) d\tilde{\theta}$$

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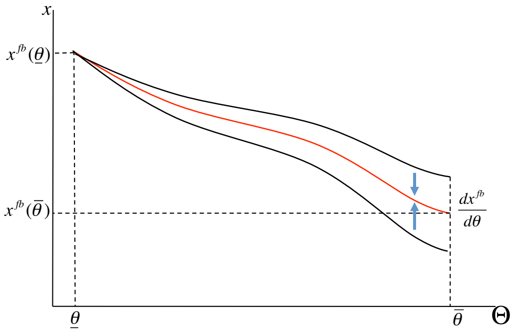


Proposition

The menu of two-part tariffs $\{p^{np}(\theta), T^{np}(\theta)\}_{\theta \in \Theta}$ *induces an output-maximizing firm to produce the first-best quantity* x^{fb}*, where*

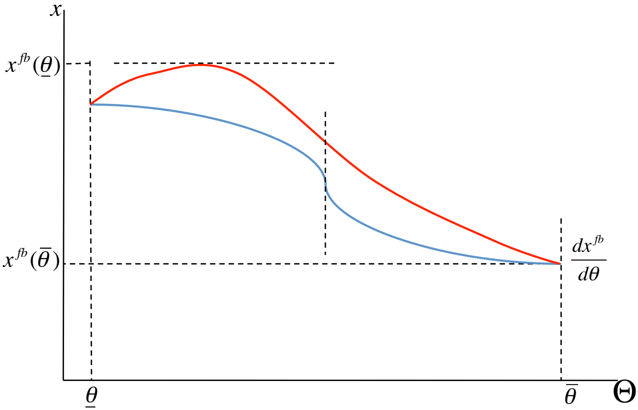
$$p^{np}(\theta) = g_x\left(x^{fb}(\theta); p^{np}(\theta), \theta\right) + \frac{g_\theta\left(x^{fb}(\theta); p^{np}(\theta), \theta\right)}{dx^{fb}(\theta)\,/d\theta},$$

$$T^{np}(\theta) = g\left(x^{fb}(\theta); p^{np}(\theta), \theta\right) - p^{np}(\theta)x^{fb}(\theta),$$

if and only if $dx^{fb}/d\theta < 0$, $dp^{np}/d\theta < 0$, *and* $p^{np}(\theta) \geq 0$ *for all* $\theta \in \Theta$*.*

Symmetric Information

Nonmarketed Good: Non-implementable Output Paths



For a marketed good, x is jointly determined by p and q.

- Regulator uses p to induce form's choice of output
- Quality level is strategically chosen based on consumer demand at p
- First-best requires p set sufficiently to induce efficient consumption.

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The regulator cannot generically induce the first-best outcome for a pure output-maximizing firm.

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Proposition

The regulator cannot generically induce the first-best outcome for a pure output-maximizing firm.

First-best output does not solve the regulator's problem, therefore it is not second-best either.

Regulator's problem

$$\max_{p,\Pi} \; V\big(x^*(p,\theta)\big) - (1+\gamma)g\big(x^*(p,\theta);\theta\big) - (\lambda+\gamma)\Pi$$

such that $\Pi \geq 0$.

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$$\max_{p,\Pi} \; V\big(x^*(p,\theta)\big) - (1+\gamma)g\big(x^*(p,\theta);\theta\big) - (\lambda+\gamma)\Pi$$

such that $\Pi \geq 0$.

Proposition

The optimal payment rule for a nonmarketed good with symmetric cost and demand information for a mixed-objectives firm consists of the unique unit price $p^{fb}_{nm}(\theta)$ and transfer payment $T^{fb}_{nm}(\theta)$ satisfying

$$p^{fb}_{nm}(\theta) = \tfrac{1}{1+\gamma} V_x\big(x^{fb}\big) + \tfrac{(1-\beta)}{\beta}\phi'\big(x^{fb}(\theta)\big),$$
$$T^{fb}_{nm}(\theta) = g\big(x^{fb},\theta\big) - p^{fb}_{nm}(\theta)x^{fb},$$

and for a pure output-maximizing firm

$$\{p^{fb}_{np}(\theta), T^{fb}_{np}(\theta)\} \in \big\{\{p,T\} \mid 0 \leq p \leq g_x(x^{so},\theta) \text{ and } T = g(x^{so},\theta) - px^{so}\big\},$$

for all $\theta \in \Theta$.

Regulator's problem is now defined as:

$$\max_{p(\theta),U(\theta)} \int_{\Theta} \left\{ V\big(x^*(p(\theta),\theta),\theta\big) - (1+\gamma) g\big(x^*(p(\theta),\theta),\theta\big) - \frac{(\lambda+\gamma)}{\beta}\big(U(\theta) - (1-\beta)\varphi(\theta)\big) \right\} dF(\theta),$$

Subject to IR and IC.

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Lemma

When the good is nonmarketed, the SCP is satisfied and $d\{U_p/U_T\}/d\theta < 0$ *for all* $\theta \in \Theta$*.*

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$$\max_{p(\theta),U(\theta)} \int_{\Theta} \{V\big(x^*(p(\theta),\theta),\theta\big) - (1+\gamma)g\big(x^*(p(\theta),\theta),\theta\big) - \tfrac{(\lambda+\gamma)}{\beta}\big(U(\theta) - (1-\beta)\varphi(\theta)\big)\}dF(\theta),$$

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Lemma

When the good is nonmarketed, the SCP is satisfied and $d\{U_p/U_T\}/d\theta < 0$ *for all* $\theta \in \Theta$*.*

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When the good is nonmarketed, the payment policy $\{p(\theta), T(\theta)\}_{\theta\in\Theta}$ *is incentive compatible only if* $dp/d\theta \leq 0$*.*

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Again the regulator's problem admits 2 cases:

1. Marginal Optimizer
2. Pure output-maximizing firm

The FOC of the Hamiltonian yields:

$$V_x(\cdot) - (1+\gamma)g_x(\cdot) = (\lambda+\gamma)\left[\frac{F(\theta)}{f(\theta)}g_{\theta x}(x^*;\theta) - \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}\right].$$

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$$p^{sb}\begin{cases} < p^{fb} & \text{when} \quad \frac{F(\theta)}{f(\theta)}d\{g_\theta(x^*;p,\theta)\}/dp > \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}, \\ = p^{fb} & \text{when} \quad \frac{F(\theta)}{f(\theta)}d\{g_\theta(x^*;p,\theta)\}/dp = \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}, \\ > p^{fb} & \text{when} \quad \frac{F(\theta)}{f(\theta)}d\{g_\theta(x^*;p,\theta)\}/dp < \frac{1-\beta}{\beta}\phi'(x^*)\frac{dx^*}{dp}. \end{cases}$$

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Proposition

When the good is nonmarketed and the firm is profit-maximizing, the second-best unit price and equilibrium quantity are distorted downward from the first-best for all but the lowest state, $\underline{\theta}$ where they are equivalent.

Recall the pricing rule for an output maximizing firm and a marketed good:

$$p^{np}(\theta) = g_x\left(x^{fb}(\theta); p^{np}(\theta), \theta\right) + \frac{g_\theta\left(x^{fb}(\theta); p^{np}(\theta), \theta\right)}{dx^{fb}(\theta)/d\theta},$$

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Proposition

When the good is nonmarketed and the firm is output-maximizing the regulator can induce the first-best outcome using the payment rule reported by Proposition 5 with the exception that $p(\theta)$ may take a negative value.

Asymmetric Demand Information

- Without quality, the source of asymmetric information changes the outcomes.
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- Likewise, asymmetric information in demand essentially creates asymmetric information in cost.
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- In consequence, the results are robust to source of asymmetric information.
- One caveat, there exists additional cases under which the second-best output is over and under-supplied

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- Recall $d\{g_\theta(x^*; p, \theta)\} / dp = g_{\theta p} + g_{\theta x}(dx^* / dp)$.

- The additional cases are a consequence of an extra term in $g_{\theta p}$
  - $g_{\theta p} = c_q q_{\theta p}$
  - $q_{\theta p} > 0$: Demand is more sensitive to the price in softer demand states.
  - $q_{\theta p} < 0$: Demand is less sensitive to the price in softer demand states.

- $g_{\theta p} < 0$

Proposition

With cost uncertainty, the relative size of the second- to first-best price and output is determined by the rules:

$$\text{(i)} \quad 0 < -\frac{g_{\theta p}}{g_{\theta x}} < \frac{dx^*}{dp} \quad \Rightarrow \frac{d}{dp}\left[\frac{\partial U}{\partial \theta}\right] > 0 \Rightarrow \begin{cases} p^{sb}(\theta) < p^{fb}(\theta) \\ x^{sb}(\theta) < x^{fb}(\theta) \end{cases}$$

$$\text{(ii)} \quad 0 < \frac{dx^*}{dp} < -\frac{g_{\theta p}}{g_{\theta x}} \quad \Rightarrow \frac{d}{dp}\left[\frac{\partial U}{\partial \theta}\right] < 0 \Rightarrow \begin{cases} p^{sb}(\theta) > p^{fb}(\theta) \\ x^{sb}(\theta) > x^{fb}(\theta) \end{cases}$$

$$\text{(iii)} \quad -\frac{g_{\theta p}}{g_{\theta x}} < 0 < \frac{dx^*}{dp} \quad \Rightarrow \frac{d}{dp}\left[\frac{\partial U}{\partial \theta}\right] < 0 \Rightarrow \begin{cases} p^{sb}(\theta) > p^{fb}(\theta) \\ x^{sb}(\theta) < x^{fb}(\theta) \end{cases}$$

for any $\theta \in (\underline{\theta}, \overline{\theta}]$ *and at* $\underline{\theta}$, $p^{sb}(\underline{\theta}) = p^{fb}(\underline{\theta})$ *and* $x^{sb}(\underline{\theta}) = x^{fb}(\underline{\theta})$.

Summary of Results

- The inclusion of quality produces qualitatively different results than when quality is absent.
- Outcome may be distorted above or below first-best when both consumers and firm are best-responding to the contract.
  - Consumer demand is responsive to price and quality creating a “double” demand-adjustment with respect to a price change.
- Removing consumers’ incentive response simplifies the regulator’s problem
  - No double-adjustment to price changes
  - Standard downward distortion in output
- Informational advantage of the firm varies with its objective and in the *best* case can be controlled

An Example

Firm:	Pure profit-maximizer
Benefit:	$B(x, q) = \log(x + q)$
Demand:	$x(q, p) = \beta + q - \alpha p^2$
Cost:	$c(x, q; \theta) = \theta x q$

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Firm's Maximizer:	$x^*(p,\theta) = \frac{1}{2\theta}(p(1-\theta\alpha p) + \theta\beta)$
Uncertainty:	$\theta \sim U[2,3]$

Let $\beta = 1$, then the second-best optimal contract gives:

Table: Price and Quantity Differences

		θ					
α		2.0	2.2	2.4	2.6	2.8	3.0
$\frac{1}{2}$	$\%\Delta p$:	0.00	-2.69	-4.72	-6.33	-7.64	-8.73
	$\%\Delta x$:	0.00	0.32	0.57	0.78	0.94	1.07
$\frac{1}{10}$	$\%\Delta p$:	0.00	-3.59	-6.15	-8.04	-9.47	-10.57
	$\%\Delta x$:	0.00	-0.65	-0.97	-1.09	-1.11	-1.07